

# Reduction and specialization of hyperelliptic continued fractions

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# Counterexample

$$\alpha = \frac{-3 + \sqrt{X^4 + 16X^2 + 24X + 9}}{X}$$

- $\text{CF}(\alpha)$  is not periodic.
- There are infinitely many primes  $p$  with

$$v_p(a_n) \geq 0 \text{ for all } n \geq 0.$$

# Example

- Choose some random example in degree 4:

$$D = X^4 + 16X^3 + 15X + 4$$
$$\text{disc } D = -87077723$$

- $D$  is not Pellian
- checked using reduction mod 3 and 5 .

# Reduction mod 13 (odd torsion order)

$n$	$\lambda(n)$	$v(\alpha_n)$	$v(a_n)$	$v(\ell c(a_n))$	$v(q_n)$	$v(\ell c(q_n))$
0	0	0	0	0	0	0
1	1	0	0	0	0	0
2	1	$-\infty$	-2	-1	-2	-1
3	2	2	2	3	2	2
4	3	-4	-4	-4	-2	-2
5	3	$-\infty$	2	3	0	1
6	4	-2	-2	-1	0	0
7	5	0	0	0	0	0
8	5	$-\infty$	-2	-1	-2	-1
9	6	2	2	3	2	2
10	7	-4	-4	-4	-2	-2
11	7	$-\infty$	2	3	0	1
12	8	-2	-2	-1	0	0
13	9	0	0	0	0	0
14	9	$-\infty$	-2	-1	-2	-1

# Reduction mod 11 (even torsion order)

$n$	$\lambda(n)$	$v(\alpha_n)$	$v(a_n)$	$v(\ell c(a_n))$	$v(q_n)$	$v(\ell c(q_n))$
0	0	0	0	0	0	0
1	1	0	0	0	0	0
2	2	0	0	0	0	0
3	2	$-\infty$	-2	-1	-2	-1
4	3	2	2	3	2	2
5	4	-4	-4	-4	-2	-2
6	5	4	4	4	2	2
7	5	$-\infty$	-6	-5	-4	-3
8	6	6	6	7	4	4
9	7	-8	-8	-8	-4	-4
10	8	8	8	8	4	4
11	8	$-\infty$	-10	-9	-6	-5
12	9	10	10	11	6	6
13	10	-12	-12	-12	-6	-6